# Neutron structures of ammonium tetrafluoroberyllate 

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#### Abstract

It is thought that hydrogen bonding is responsible for the ferroelectricity in ammonium tetrafluoroberyllate, $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{BeF}_{4}$. In the past X-ray data have been collected, but these did not permit accurate determination of the H -atom positions. In order to obtain more accurate information the neutron structures have now been determined for the paraelectric and ferroelectric phases. Going from the paraelectric to the ferroelectric phase, both the $\mathrm{BeF}_{4}^{2-}$ and the $\mathrm{NH}_{4}^{+}$ions rotate and shift from the mirror planes of the paraelectric phase. This results in removal of the mirror-plane symmetry and formation of a superlattice with the $a$ axis doubled. Along the polar $c$ axis, the $\mathrm{NH}_{4}^{+}$ions move towards the $\mathrm{BeF}_{4}^{2-}$ ions within chains of molecules and the chains move slightly relative to one another. The rotations and translations give rise to stronger hydrogen-bonding interactions.


## 1. Introduction

The room-temperature phase of ammonium tetrafluoroberyllate (AFB) is paraelectric with space group Pnma. AFB undergoes a nonferroelectric transition at 182 K (Strukov et al., 1973; Makita \& Yamauchi, 1974) to a phase that is incommensurate along the $a$ axis with a period of $\sim 2 a$ (Iizumi \& Gesi, 1977). A ferroelectric transition occurs at 175 K (Pepinsky \& Jona, 1957). In the ferroelectric phase the $a$ axis is doubled compared with the paraelectric phase and the space group is $P n a 2_{1}$. Several investigators have measured physical properties, including spontaneous polarization (Hoshino et al., 1958), dielectric behavior (Strukov et al., 1962; Ohshima \& Nakamura, 1966) and changes in activation energies through the phase transition (O'Reilly et al., 1967). An improper ferroelectric model has been proposed (Levanyuk \& Sannikov, 1969), but the mechanism of the phase transition is not clear. O'Reilly et al. (1967) suggest that one of the two independent ammonium ions in the unit cell becomes ordered in the ferroelectric phase and the transition is of order-disorder type. However, deuterated AFB samples show almost no change in the activation energies of ammonium ions on going through the phase transition (Kydon et al., 1969). Onodera \& Shiozaki (1979) report some disorder in one of the two independent ammonium ions in the para-
electric phase of AFB and also fairly dissimilar distortions in the four symmetry-nonequivalent ammonium ions in the structure at 133 K after going through the ferroelectric phase transition. They also suggest that the two ammonium ions that result from the ordering of the ion that they find to be disordered in the paraelectric phase take antiparallel up and down states along the ferroelectric polar axis with the consequent loss of the mirror symmetry. This occurs because of stronger hydrogen bonding and resulting restricted freedom in the ferroelectric phase. They do, however, suggest more accurate dielectric and diffraction experiments. According to Garg \& Srivastava (1979), the X-ray crystal structure of AFB in the paraelectric phase does not indicate any disorder in the ammonium ions. These authors, like Onodera \& Shiozaki (1979), conclude that ferroelectricity in AFB most probably results from changes in hydrogen bonding. In order to obtain more accurate H -atom positions, the neutron structures have now been determined for the paraelectric and ferroelectric phases.

## 2. Experimental

From a large crystal (approximately $1.5 \times 1.0 \times 0.5 \mathrm{~cm}$ ), provided by Professor Y. Ono of Tohoku University, Japan, a piece was cut and mounted on an aluminium pin with halocarbon grease (Dow-Corning), sealed in an aluminium container under a helium atmosphere, and placed in a DISPLEX Model CS-202 closed-cycle refrigerator (APD Cryogenics Inc.). Neutron diffraction data were obtained on the four-circle diffractometer at beam port H6M of the High Flux Beam Reactor at Brookhaven National Laboratory at 200, 163 and 20 K. The neutron beam, monochromated by $\operatorname{Be}(002)$ planes in reflection geometry, had a wavelength of 1.0462 (1) $\AA$, as calibrated against a KBr crystal $\left(a_{o}=6.6000 \mathrm{~A}\right.$ at $295 \mathrm{~K})$. The intensities of two reflections were monitored periodically as a check on the experimental stability and these showed no systematic variation throughout. The crystal temperature was maintained within $\pm 0.5 \mathrm{~K}$ during each experiment. The unit cells were determined by a least-squares fit of $\sin ^{2} \theta$ values for 32 reflections in the range $20<2 \theta<27^{\circ}$. Intensity data were measured over one octant by means of $\theta / 2 \theta$ step

Table 1. Experimental details

|  | 200 K | 163 K | 20 K |
| :---: | :---: | :---: | :---: |
| Crystal data |  |  |  |
| Chemical formula | $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{BeF}_{4}$ | $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{BeF}_{4}$ | $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{BeF}_{4}$ |
| Chemical formula weight | 121.093 | 121.093 | 121.093 |
| Cell setting | Orthorhombic | Orthorhombic | Orthorhombic |
| Space group | Pnma | Pna2 ${ }_{1}$ | Pna2 ${ }_{1}$ |
| $a$ ( A ) | 7.531 (3) | 15.017 (6) | 14.997 (5) |
| $b$ ( ${ }_{\text {® }}$ ) | 5.874 (2) | 5.876 (3) | 5.860 (3) |
| $c(\AA)$ | 10.399 (4) | 10.418 (4) | 10.402 (4) |
| $V\left(\AA^{3}\right)$ | 460.0 (5) | 919.2 (12) | 914.1 (11) |
| $Z$ | 4 | 8 | 8 |
| $D_{x}\left(\mathrm{Mg} \mathrm{m}^{-3}\right)$ | 1.748 | 1.750 | 1.759 |
| Radiation type | Neutron | Neutron | Neutron |
| Wavelength (A) | 1.0462 | 1.0462 | 1.0462 |
| No. of reflections for cell parameters | 32 | 32 | 32 |
| $\theta$ range ( ${ }^{\circ}$ ) | 20-27 | 20-27 | 20-27 |
| $\mu\left(\mathrm{mm}^{-1}\right)$ | 0.291 | 0.291 | 0.293 |
| Temperature (K) | 200 | 163 | 20 |
| Crystal form | Box | Box | Box |
| Crystal size (mm) | $4.0 \times 3.9 \times 3.9$ | $4.0 \times 3.9 \times 3.9$ | $4.0 \times 3.9 \times 3.9$ |
| Crystal color | Colorless | Colorless | Colorless |
| Data collection |  |  |  |
| Diffractometer | Beamline H6M, HFBR | Beamline H6M, HFBR | Beamline H6M, HFBR |
| Data collection method | $\theta / 2 \theta$ scans | $\theta / 2 \theta$ scans | $\theta / 2 \theta$ scans |
| Absorption correction | Analytical | Analytical | Analytical |
| $T_{\text {min }}$ | 0.356 | 0.346 | 0.343 |
| $T_{\text {max }}$ | 0.428 | 0.432 | 0.431 |
| No. of measured reflections | 1018 | 1972 | 1516 |
| No. of independent reflections | 996 | 1932 | 1484 |
| No. of observed reflections | 790 | 1390 | 1363 |
| Criterion for observed reflections | $I>3 \sigma(I)$ | $I>3 \sigma(I)$ | $I>3 \sigma(I)$ |
| $\theta_{\text {max }}\left({ }^{\circ}\right.$ ) | 55 | 55 | 55 |
| Range of $h, k, l$ | $0 \rightarrow h \rightarrow 11$ | $0 \rightarrow h \rightarrow 23$ | $0 \rightarrow h \rightarrow 23$ |
|  | $0 \rightarrow k \rightarrow 9$ | $0 \rightarrow k \rightarrow 9$ | $0 \rightarrow k \rightarrow 5$ |
|  | $0 \rightarrow l \rightarrow 16$ | $0 \rightarrow l \rightarrow 16$ | $0 \rightarrow l \rightarrow 16$ |
| No. of standard reflections | 2 | 2 | 2 |
| Frequency of standard reflections | Every 100 reflections | Every 100 reflections | Every 100 reflections |
| Intensity decay (\%) | 0 | 0 | 0 |
| Refinement |  |  |  |
| Refinement on | $F^{2}$ | $F^{2}$ | $F^{2}$ |
| $R\left(F^{2}\right)$ | 0.0679 | 0.0476 | 0.0342 |
| $w R\left(F^{2}\right)$ | 0.0733 | 0.0552 | 0.0405 |
| $S$ | 1.0428 | 1.0650 | 1.0436 |
| No. of reflections used in refinement | 996 | 1932 | 1484 |
| No. of parameters used | 83 | 271 | 271 |
| H -atom treatment | Refined | Refined | Refined |
| Weighting scheme | $w=1 /\left[\sigma^{2}(F)+0.0004 F^{2}\right]$ | $w=1 /\left[\sigma^{2}(F)+0.0004 F^{2}\right]$ | $w=1 /\left[\sigma^{2}(F)+0.0004 F^{2}\right]$ |
| $(\Delta / \sigma)_{\max }$ | 0.01 | 0.01 | 0.01 |
| Extinction method | Type I, isotropic | Type I, isotropic | Type I, isotropic |
| Extinction coefficient | $36(2) \times 10^{2}$ | $39(1) \times 10^{2}$ | $51(1) \times 10^{2}$ |
| Source of atomic scattering factors | International Tables for Crystallography (1995, Vol. C) | International Tables for Crystallography (1995, Vol. C) | International Tables for Crystallography (1995, Vol. C) |

scans. Integrated intensities $I_{o}$ and variances $\sigma^{2}\left(I_{o}\right)$ were derived from the scan profiles as described previously (McMullan et al., 1979). Lorentz factors were applied, as well as an analytical absorption correction (de Meulenaer \& Tompa, 1965; Templeton \& Templeton, 1973). The structure model was refined based on all independent $F_{o}^{2}$ values using $U P A L S$ (Lundgren, 1982). The
atomic positions determined by X-ray diffraction $\dagger$ were used as a starting structure. The scale factor was varied,

[^0]Table 2. Fractional atomic coordinates

|  | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| 200 K |  |  |  |
| N1 | 0.1850 (2) | 1/4 | 0.0990 (1) |
| N2 | 0.4610 (2) | 1/4 | 0.8044 (1) |
| Be | 0.2505 (2) | 1/4 | 0.4184 (1) |
| F1 | 0.0531 (3) | 1/4 | 0.3817 (3) |
| F2 | 0.2747 (3) | 1/4 | 0.5639 (2) |
| F3 | 0.3354 (2) | 0.0348 (3) | 0.3638 (2) |
| H1 | 0.3137 (7) | 1/4 | 0.1244 (6) |
| H2 | 0.0996 (9) | 1/4 | 0.1711 (7) |
| H3 | 0.1639 (6) | 0.1137 (9) | 0.0497 (7) |
| H4 | 0.4897 (10) | 1/4 | 0.7106 (5) |
| H5 | 0.5686 (7) | 1/4 | 0.8596 (7) |
| H6 | 0.3854 (8) | 0.1174 (8) | 0.8238 (4) |
| 163 K |  |  |  |
| N11 | 0.2188 (2) | 0.2600 (3) | 0.0990 |
| N12 | 0.4691 (2) | 0.2527 (2) | 0.3978 (1) |
| N21 | 0.3534 (2) | 0.2562 (2) | 0.8015 (3) |
| N22 | 0.6025 (2) | 0.2404 (3) | 0.6957 (3) |
| Be1 | 0.2509 (2) | 0.2536 (3) | 0.4161 (3) |
| Be 2 | 0.5008 (2) | 0.2553 (3) | 0.0785 (4) |
| F11 | 0.1534 (3) | 0.2299 (6) | 0.3769 (6) |
| F12 | 0.2584 (3) | 0.2867 (6) | 0.5617 (4) |
| F13 | 0.2989 (2) | 0.0333 (5) | 0.3792 (5) |
| F14 | 0.2903 (2) | 0.4609 (5) | 0.3490 (4) |
| F21 | 0.4026 (3) | 0.2750 (7) | 0.1231 (6) |
| F22 | 0.5084 (3) | 0.2235 (6) | -0.0674 (4) |
| F23 | 0.5485 (3) | 0.4767 (6) | 0.1175 (5) |
| F24 | 0.5417 (3) | 0.0452 (6) | 0.1443 (5) |
| H111 | 0.2820 (6) | 0.2840 (15) | 0.1285 (10) |
| H112 | 0.1773 (6) | 0.2828 (14) | 0.1732 (10) |
| H113 | 0.2137 (6) | 0.0995 (12) | 0.0681 (11) |
| H114 | 0.2050 (4) | 0.3645 (10) | 0.0218 (6) |
| H121 | 0.5342 (4) | 0.2658 (11) | 0.3743 (10) |
| H122 | 0.4267 (7) | 0.2783 (16) | 0.3255 (11) |
| H123 | 0.4604 (4) | 0.0883 (9) | 0.4239 (7) |
| H124 | 0.4546 (5) | 0.3666 (15) | 0.4646 (10) |
| H211 | 0.3720 (5) | 0.2607 (17) | 0.7123 (7) |
| H212 | 0.4091 (6) | 0.2233 (15) | 0.8513 (10) |
| H213 | 0.3032 (3) | 0.1444 (9) | 0.8207 (6) |
| H214 | 0.3257 (7) | 0.4022 (14) | 0.8282 (8) |
| H221 | 0.6125 (7) | 0.2383 (12) | 0.7935 (8) |
| H222 | 0.6563 (6) | 0.2190 (15) | 0.6382 (10) |
| H223 | 0.5583 (8) | 0.1183 (17) | 0.6792 (9) |
| H224 | 0.5849 (5) | 0.4021 (10) | 0.6796 (8) |


| 20 K |  |  |  |
| :--- | :--- | :--- | ---: |
| N11 | $0.2196(1)$ | $0.2666(3)$ | 0.0990 |
| N12 | $0.4712(1)$ | $0.2538(2)$ | $0.3941(1)$ |
| N21 | $0.3510(1)$ | $0.2600(2)$ | $0.7972(2)$ |
| N22 | $0.6007(1)$ | $0.2369(2)$ | $0.6937(2)$ |
| Be1 | $0.2517(2)$ | $0.2578(2)$ | $0.4152(2)$ |
| Be2 | $0.5029(1)$ | $0.2574(2)$ | $0.0774(2)$ |
| F11 | $0.1544(2)$ | $0.2247(4)$ | $0.3718(3)$ |
| F12 | $0.2565(1)$ | $0.2991(5)$ | $0.5624(3)$ |
| F13 | $0.3028(1)$ | $0.0366(3)$ | $0.3832(3)$ |
| F14 | $0.2905(1)$ | $0.4680(4)$ | $0.3475(3)$ |
| F21 | $0.4054(2)$ | $0.2905(5)$ | $0.1265(3)$ |
| F22 | $0.5049(2)$ | $0.2173(4)$ | $-0.0687(3)$ |
| F23 | $0.5540(1)$ | $0.4773(4)$ | $0.1097(3)$ |
| F24 | $0.5421(1)$ | $0.0466(4)$ | $0.1451(3)$ |
| H111 | $0.2832(3)$ | $0.2959(10)$ | $0.1288(6)$ |
| H112 | $0.1754(4)$ | $0.2971(11)$ | $0.1708(5)$ |
| H113 | $0.2151(3)$ | $0.0991(10)$ | $0.0735(5)$ |
| H114 | $0.2054(3)$ | $0.3706(8)$ | $0.0213(4)$ |

Table 2 (cont.)

|  | $x$ | $y$ | $z$ |
| :--- | :--- | :--- | :--- |
| H121 | $0.5369(3)$ | $0.2753(8)$ | $0.3712(6)$ |
| H122 | $0.4319(3)$ | $0.2792(9)$ | $0.3164(5)$ |
| H123 | $0.4622(3)$ | $0.0894(9)$ | $0.4226(5)$ |
| H124 | $0.4534(2)$ | $0.3611(7)$ | $0.4672(4)$ |
| H211 | $0.3634(5)$ | $0.2620(7)$ | $0.7056(5)$ |
| H212 | $0.4066(3)$ | $0.2192(9)$ | $0.8479(5)$ |
| H213 | $0.2997(2)$ | $0.1500(8)$ | $0.8156(4)$ |
| H214 | $0.3332(3)$ | $0.4222(8)$ | $0.8244(4)$ |
| H221 | $0.6183(4)$ | $0.2235(10)$ | $0.7903(6)$ |
| H222 | $0.6563(3)$ | $0.1994(11)$ | $0.6397(5)$ |
| H223 | $0.5494(2)$ | $0.1289(7)$ | $0.6742(4)$ |
| H224 | $0.5835(3)$ | $0.4013(9)$ | $0.6752(5)$ |

together with positional and anisotropic displacement parameters for all atoms and an isotropic type I extinction parameter (Becker \& Coppens, 1974). Scattering lengths used in the refinements were N 9.36 , Be 7.79, F 5.654 and H -3.7406 fm (Sears, 1995). The final difference Fourier map did not show significant positive or negative peaks at any of the three temperatures. The experimental data are listed in Table 1 and fractional atomic coordinates in Table 2.

## 3. Results and discussion

The bond lengths and angles of the $\mathrm{BeF}_{4}^{2-}$ ions from the three neutron determinations are listed in Table 3. The $\mathrm{Be}-\mathrm{F}$ bond lengths at 200 K vary from 1.524 (3) to 1.535 (3) $\AA$ and the F-Be-F angles vary from 108.2 (1) to $111.3(2)^{\circ}$. The corresponding values for the roomtemperature X-ray structure (Garg \& Srivastava, 1979) are very similar: 1.522 (6) -1.535 (4) $\AA$ and 108.4 (2)$111.0(5)^{\circ}$. Both these tetrahedra are quite regular with mean sides $1.531 \AA$ and mean angles $109.5^{\circ}$. These values also agree well with those from other crystal structures involving $\mathrm{BeF}_{4}^{2-}$ ions (Vicat et al., 1974; Le Roy \& Aleonard, 1972).

In the ferroelectric phase the nonequivalent $\mathrm{BeF}_{4}^{2-}$ tetrahedra become somewhat less regular. If the differences between the maximum and minimum bond lengths and angles of these tetrahedra are defined as $\Delta_{l}$ and $\Delta_{a}$, respectively, then at 163 K , for ${\mathrm{Be} 1 \mathrm{~F}_{4}, \Delta_{l}=}$, $0.009 \AA$ and $\Delta_{a}=4.3^{\circ}$; for $\mathrm{Be} 2 \mathrm{~F}_{4}, \Delta_{l}=0.014 \AA$ and $\Delta_{a}=$ $4.6^{\circ}{ }^{\circ} \dagger$ These may be compared with the corresponding values (X-ray structure at 133 K ) of Onodera \& Shiozaki (1979): for $\mathrm{Be} 1 \mathrm{~F}_{4}, \Delta_{l}=0.088 \AA$ [1.507 (6)1.595 (6) $\AA$ ] and $\Delta_{a}=10.2^{\circ}\left[105.5\right.$ (3)-115.7 (4) $\left.{ }^{\circ}\right]$; for $\mathrm{Be} 2 \mathrm{~F}_{4}, \Delta_{l}=0.063 \AA[1.513$ (6)-1.576 (6) $\AA]$ and $\Delta_{a}=$ $6.7^{\circ}$ [106.8 (3)-113.5 (3) ${ }^{\circ}$ ]. The distortions here appear larger and dissimilar for the two ions. In our 20 K
 $\mathrm{Be} 2 \mathrm{~F}_{4}$ the corresponding values are $0.029 \AA$ and $5.3^{\circ}$, which are similar to those obtained at 163 K .

[^1]The bond lengths and angles for the ammonium ions from the three neutron determinations are included in Table 3. In the 200 K structure, for the ${\mathrm{N} 1 \mathrm{H}_{4} \text { ion the }}^{\text {in }}$ shortest and longest $\mathrm{N}-\mathrm{H}$ lengths are 0.964 (6) and 1.004 (5) $\AA$, and the smallest and largest $\mathrm{H}-\mathrm{N}-\mathrm{H}$ angles are 107.3 (4) and 115.4 (6) ${ }^{\circ}$, while for the $\mathrm{N} 2 \mathrm{H}_{4}$ ion the corresponding bond lengths are 0.985 (6) and 1.000 (5) $\AA$, and the angles are 108.9 (4) and 112.8 (6) ${ }^{\circ}$. In the room-temperature structure determination by Garg \& Srivastava (1979), as well as the structure by Onodera \& Shiozaki (1979), the bond lengths vary from 0.72 to $1.01 \AA$ and the bond angles from 101 to $123^{\circ}$. However, the mean $\mathrm{N}-\mathrm{H}$ bond length and mean $\mathrm{H}-$ $\mathrm{N}-\mathrm{H}$ angle in both cases are $0.87 \AA$ and $109^{\circ}$, respectively. The H atoms have been assigned displacement parameters of the associated N atoms in both determinations and thus these are not of much significance. Onodera \& Shiozaki (1979), however, considered one ammonium ion to be somewhat disordered, because the corresponding electron distribution is more diffuse. The mean $\mathrm{H}-\mathrm{N}-\mathrm{H}$ angle found from the neutron determination is $110.4^{\circ}$ and the $\mathrm{N}-\mathrm{H}$ length is $0.989 \AA$, which is more than $0.1 \AA$ longer than in the X-ray structures. This is in good agreement with the $\mathrm{N}-\mathrm{H}$ length of $0.97 \AA$ and $\mathrm{H}-\mathrm{N}-\mathrm{H}$ angle of $109.3^{\circ}$ reported by Schlemper \& Hamilton (1966) from their neutron structure of ammonium sulfate. $\dagger$ No indication of disorder was seen in the H -atom positions in the present work.

At 163 K the four $\mathrm{NH}_{4}^{+}$ions have an overall mean $\mathrm{N}-\mathrm{H}$ bond length of $1.004 \AA$ (maximum s.u. $=0.011 \AA$ ); the mean angle is $109.4^{\circ}$ (maximum s.u. $=0.9^{\circ}$ ). The shortest and longest $\mathrm{N}-\mathrm{H}$ bond lengths are 0.971 (8) and 1.033 (6) $\AA$; the variation in bond angles is 105.0 (8)-118.6 (8) ${ }^{\circ}$. At 20 K the neutron structure gives an overall mean $\mathrm{N}-\mathrm{H}$ distance of $1.018 \AA$ (maximum s.u. $=0.007 \AA$ ); the mean angle is $109.5^{\circ}$ (maximum s.u. $=$

[^2]Table 3. Selected geometric parameters ( $\AA,^{\circ}$ )

|  | 200 K | 163 K | 20 K |
| :---: | :---: | :---: | :---: |
| Be1-F11 | 1.535 (3) | 1.526 (6) | 1.539 (3) |
| Be1-F12 | 1.524 (3) | 1.533 (6) | 1.553 (4) |
| Be1-F13 | 1.526 (2) | 1.531 (4) | 1.542 (3) |
| Be1-F14 |  | 1.524 (4) | 1.533 (3) |
| Be2-F21 |  | 1.550 (6) | 1.560 (3) |
| Be2-F22 |  | 1.536 (6) | 1.539 (4) |
| Be2-F23 |  | 1.540 (5) | 1.537 (3) |
| Be2-F24 |  | 1.539 (4) | 1.538 (3) |
| N11-H111 | 1.004 (5) | 1.008 (9) | 1.018 (5) |
| N11-H112 | 0.988 (7) | 1.002 (10) | 1.014 (6) |
| N11-H113 | 0.964 (6) | 0.999 (8) | 1.019 (6) |
| N11-H114 |  | 1.033 (6) | 1.033 (5) |
| N12-H121 |  | 1.012 (7) | 1.020 (5) |
| N12-H122 |  | 0.997 (11) | 1.012 (6) |
| N12-H123 |  | 1.012 (6) | 1.017 (5) |
| N12 - H124 |  | 0.989 (10) | 1.022 (5) |
| N21-H211 | 1.000 (5) | 0.971 (8) | 0.971 (6) |
| N21-H212 | 0.993 (6) | 1.003 (10) | 1.015 (5) |
| N21-H213 | 0.985 (6) | 1.020 (5) | 1.023 (4) |
| N21-H214 |  | 0.993 (9) | 1.027 (5) |
| N22-H221 |  | 1.030 (9) | 1.043 (7) |
| N22-H222 |  | 1.012 (10) | 1.028 (5) |
| N22-H223 |  | 0.993 (11) | 1.017 (4) |
| N22-H224 |  | 1.001 (6) | 1.016 (5) |
| F11-Be1-F12 | 111.3 (2) | 110.3 (3) | 110.6 (2) |
| F11-Be1-F13 | 108.2 (1) | 107.9 (3) | 107.6 (2) |
| F11-Be1-F14 | 108.6 (1) | 108.8 (3) | 109.1 (2) |
| F12-Be1-F13 |  | 108.7 (3) | 108.7 (2) |
| F12-Be1-F14 |  | 108.9 (3) | 108.1 (2) |
| F13-Be1-F14 |  | 112.2 (3) | 112.8 (2) |
| F21-Be2-F22 |  | 112.1 (3) | 111.1 (2) |
| F21-Be2-F23 |  | 107.5 (3) | 107.0 (2) |
| F21-Be2-F24 |  | 107.8 (3) | 108.0 (2) |
| F22-Be2-F23 |  | 108.3 (3) | 109.5 (2) |
| F22-Be2-F24 |  | 109.2 (3) | 108.8 (2) |
| F23-Be2-F24 |  | 112.0 (3) | 112.5 (2) |
| H111-N11-H112 | 115.4 (6) | 109.4 (8) | 111.0 (5) |
| H111 - N11 - H113 | 107.4 (3) | 107.6 (7) | 107.6 (4) |
| H111 - N11 - H114 | 107.3 (4) | 110.1 (7) | 109.1 (4) |
| H112 - N11-H113 |  | 109.1 (8) | 108.6 (5) |
| H112 - N11-H114 |  | 113.3 (7) | 110.0 (4) |
| H113-N11-H114 |  | 107.1 (7) | 110.6 (4) |
| H121 - N12-H122 |  | 115.0 (8) | 111.0 (4) |
| H121 - N12-H123 |  | 105.2 (5) | 108.2 (4) |
| H121 - N12-H124 |  | 109.3 (7) | 110.4 (4) |
| H122 - N12-H123 |  | 105.4 (7) | 107.2 (4) |
| H122 - N12-H124 |  | 106.8 (8) | 110.6 (4) |
| H123 - N12-H124 |  | 115.3 (7) | 109.3 (4) |
| H211-N21-H212 | 112.8 (6) | 105.1 (8) | 110.8 (6) |
| H211-N21-H213 | 108.9 (4) | 114.7 (7) | 109.6 (5) |
| H211-N21-H214 | 110.7 (4) | 111.4 (8) | 108.0 (4) |
| H212-N21-H213 |  | 113.0 (6) | 111.9 (4) |
| H212-N21-H214 |  | 111.8 (8) | 106.8 (4) |
| H213-N21-H214 |  | 101.1 (7) | 109.6 (4) |
| H221-N22-H222 |  | 117.9 (9) | 107.8 (5) |
| H221 - N22-H223 |  | 105.0 (8) | 109.7 (4) |
| H221 - N22-H224 |  | 102.4 (7) | 108.6 (4) |
| H222-N22-H223 |  | 110.0 (8) | 111.8 (4) |
| H222-N22-H224 |  | 103.3 (7) | 107.8 (5) |
| H223-N22-H224 |  | 118.6 (8) | 111.1 (4) |

$0.6^{\circ}$ ) for all four ions. The shortest and longest $\mathrm{N}-\mathrm{H}$ bond lengths are 0.971 (6) and 1.043 (7) $\AA$; the variation in angles is $107.2(4)-111.9(4)^{\circ}$. It may be noted that the
angular distortions at 20 K are less pronounced than at 163 K and the $\mathrm{N}-\mathrm{H}$ bond lengths are slightly, but systematically, increased at lower temperatures. This is to be expected, as none of the bond lengths reported above are corrected for the anisotropic thermal motion of the atoms.


Fig. 1. Representation down the $b$ axis (a) at 200 K , (b) at 163 K and (c) at 20 K . The view is from infinity, so that one $\mathrm{H}(\mathrm{F})$ atom of each ion is hidden. Only the ions near the plane $y=0.25$ are shown. The unit cell at 163 and 20 K is shifted by $(0.1250 .00 .0)$ to correspond to the unit cell at 200 K . Ellipsoids are shown at $50 \%$ probability (Johnson, 1976). The larger ellipsoids correspond to the H atoms of the cations.

The structures in the paraelectric phase ( 200 K ) and the ferroelectric phase ( 163 and 20 K ), as viewed down the $b$ axis, are shown in Fig. 1. This figure also shows the large anisotropic thermal motion of atoms at 200 K , which decreases upon cooling and becomes quite small at 20 K . One of the H atoms that lies near $y=\frac{1}{4}, \mathrm{H} 211$,

(a)

(b)

Fig. 2. View down the $c$ axis $(a)$ at 200 K and $(b)$ at 163 K , showing the shifts from the mirror planes $y=0.25$ and 0.75 . The unit cell at 163 K is shifted by $(0.1250 .00 .0)$ to correspond to the unit cell at 200 K . The N atoms are shaded.
has an unusually large $U^{22}$ value at 163 and 20 K . Figs. $2(a)$ and $2(b)$ show the structures in the paraelectric and ferroelectric phases as viewed down the $c$ axis. Thermal motion has been omitted for clarity in these latter figures. It will be noticed that on going through the phase transition both the $\mathrm{BeF}_{4}^{2-}$ and $\mathrm{NH}_{4}^{+}$tetrahedra rotate as well as shift from the mirror planes at $y=\frac{1}{4}$ and $\frac{3}{4}$ of the paraelectric phase. This twist in the tetrahedra causes distortions that are more pronounced for $\mathrm{NH}_{4}^{+}$ ions than $\mathrm{BeF}_{4}^{2-}$ ions. The twist and distortions give rise to stronger hydrogen-bonding interactions and stabilize the structure. Any contributions to the distortions that are due to anisotropy of the thermal motion will be diminished at lower temperatures, away from the transition temperature, e.g. 20 K . The distortions present here are mainly due to the requirements of the hydrogen-bonding scheme and hence are less severe than at 163 K , which is below, but close to, the transition temperature.

Fig. 3 shows a hydrogen-bonding scheme in which an attempt has been made to link all F atoms. This requires an $\mathrm{H} \cdots \mathrm{F}$ distance limit of $2.45 \AA$. Obviously, many of these interactions are very weak. To consider only stronger interactions, $\mathrm{H} \cdots \mathrm{F}<2.1 \AA$ may be chosen as the criterion. This is almost equivalent to the van der Waals contact-distance cutoff of $\mathrm{N} \cdots \mathrm{F}<2.85 \AA$ introduced by Pimentel \& McClellan (1960). The strong bonds are indicated with open lines in Fig. 3 and the others by thin solid lines. At 200 K , in the paraelectric phase, there is one such stronger bond for $\mathrm{N} 1 \mathrm{H}_{4}$ and two for $\mathrm{N} 2 \mathrm{H}_{4}$. At 163 K , there are four such bonds for the $\mathrm{N} 22 \mathrm{H}_{4}$ ion and three for each of the other three ammonium ions. At 20 K , due to a small rotation, one of these bonds involving $\mathrm{N} 21 \mathrm{H}_{4}$ falls slightly above the $2.1 \AA$ cutoff. These stronger hydrogen bonds at all three temperatures are listed in Table 4.

The difference between the paraelectric phase and the ferroelectric phase occurs mainly along the polar $c$ axis. In the ferroelectric phase the ammonium ions move $\sim 0.03 \AA$ closer to the nearest tetrafluoroberyllate ions within chains of molecules. The chains themselves move slightly relative to one another. The Be...Be distances in this way are increased by $\sim 0.010 \AA$. According to


Fig. 3. Stereoview down the $b$ axis, with the origin at the lower left and the $a$ axis pointing to the right, showing hydrogen bonding in AFB at 163 K . Hydrogen bonds with $\mathrm{H} \cdots \mathrm{F}<2.10 \mathrm{~A}$, also listed in Table 4, are indicated by double lines. Weaker interactions, with $2.10 \AA<$ $\mathrm{H} \cdots \mathrm{F}<2.45 \AA$, are indicated by single lines.

Table 4. Hydrogen-bonding geometry $\left(\AA,^{\circ}\right)$
200 K

| $D-\mathrm{H} \cdots A$ | $D-\mathrm{H}$ | $\mathrm{H} \cdots A$ | $D \cdots A$ | $D-\mathrm{H} \cdots A$ |
| :--- | :---: | :--- | :--- | :--- |
| $\mathrm{~N} 1-\mathrm{H} 1 \cdots \mathrm{~F}^{\mathrm{i}}$ | $1.004(5)$ | $1.804(6)$ | $2.779(2)$ | $162.7(6)$ |
| $\mathrm{N} 2-\mathrm{H} 5 \cdots \mathrm{~F}^{\text {ii }}$ | $0.993(6)$ | $1.744(6)$ | $2.731(3)$ | $171.9(6)$ |
| $\mathrm{N} 2-\mathrm{H} 6 \cdots \mathrm{~F}^{\text {iii }}$ | $0.986(5)$ | $1.933(6)$ | $2.857(2)$ | $155.0(5)$ |
| Symmetry codes: | (i) $\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z ;$ | (ii) $\frac{1}{2}+x, y, \frac{3}{2}-z ; \quad$ (iii) |  |  |
| $\frac{1}{2}-x,-y, \frac{1}{2}+z$. |  |  |  |  |

163 K

| $D-\mathrm{H} \cdots A$ | $D-\mathrm{H}$ | $\mathrm{H} \cdots A$ | $D \cdots A$ | $D-\mathrm{H} \cdots A$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~N} 11-\mathrm{H} 111 \cdots \mathrm{~F} 21$ | $1.008(9)$ | $1.813(10)$ | $2.774(5)$ | $158.0(9)$ |
| $\mathrm{N} 11-\mathrm{H} 113 \cdots \mathrm{~F} 12^{\text {i }}$ | $0.999(8)$ | $1.887(8)$ | $2.829(4)$ | $156.0(9)$ |
| $\mathrm{N} 11-\mathrm{H} 114 \cdots \mathrm{~F} 13^{\text {ii }}$ | $1.033(6)$ | $1.787(8)$ | $2.810(5)$ | $169.6(5)$ |
| $\mathrm{N} 12-\mathrm{H} 121 \cdots \mathrm{~F} 11^{\text {iii }}$ | $1.011(7)$ | $1.790(8)$ | $2.779(5)$ | $164.7(9)$ |
| $\mathrm{N} 12-\mathrm{H} 123 \cdots \mathrm{~F} 22^{\text {iv }}$ | $1.012(6)$ | $1.893(7)$ | $2.842(4)$ | $154.8(6)$ |
| $\mathrm{N} 12-\mathrm{H} 124 \cdots \mathrm{~F} 23^{\text {v }}$ | $0.989(10)$ | $1.840(11)$ | $2.799(5)$ | $162.3(8)$ |
| $\mathrm{N} 21-\mathrm{H} 212 \cdots \mathrm{~F} 22^{\text {vi }}$ | $1.003(10)$ | $1.715(10)$ | $2.705(5)$ | $168.6(8)$ |
| $\mathrm{N} 21-\mathrm{H} 213 \cdots \mathrm{~F} 14^{\text {vii }}$ | $1.020(5)$ | $1.795(6)$ | $2.812(4)$ | $176.1(5)$ |
| $\mathrm{N} 21-\mathrm{H} 214 \cdots \mathrm{~F} 11^{\text {viii }}$ | $0.993(9)$ | $2.016(9)$ | $2.894(4)$ | $146.2(9)$ |
| $\mathrm{N} 21-\mathrm{H} 214 \cdots \mathrm{~F} 13^{\text {viii }}$ | $0.993(9)$ | $2.092(11)$ | $2.921(4)$ | $139.8(8)$ |
| $\mathrm{N} 22-\mathrm{H} 222 \cdots \mathrm{~F} 12^{\text {iii }}$ | $1.012(10)$ | $1.728(10)$ | $2.730(5)$ | $169.1(9)$ |
| $\mathrm{N} 22-\mathrm{H} 223 \cdots \mathrm{~F} 24^{\text {iv }}$ | $0.993(11)$ | $1.819(11)$ | $2.792(4)$ | $165.7(9)$ |
| $\mathrm{N} 22-\mathrm{H} 224 \cdots \mathrm{~F} 21^{\text {v }}$ | $1.001(6)$ | $1.996(7)$ | $2.948(4)$ | $158.0(7)$ |

Symmetry codes: (i) $\frac{1}{2}-x, y-\frac{1}{2}, z-\frac{1}{2}$; (ii) $\frac{1}{2}-x, \frac{1}{2}+y, z-\frac{1}{2}$; (iii) $\frac{1}{2}+x, \frac{1}{2}-y, z$; (iv) $1-x,-y, \frac{1}{2}+z$; (v) $1-x, 1-y, \frac{1}{2}+z$; (vi) $x, y, 1+z$; (vii) $\frac{1}{2}-x, y-\frac{1}{2}, \frac{1}{2}+z$; (viii) $\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$.

| 20 K |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $D-\mathrm{H} \cdots A$ | $D-\mathrm{H}$ | H. . $A$ | D..A | $D-\mathrm{H} \cdots A$ |
| N11-H111...F21 | 1.018 (9) | 1.833 (5) | 2.805 (3) | 158.5 (5) |
| N11-H113...F12 ${ }^{\text {i }}$ | 1.019 (8) | 1.813 (7) | 2.789 (4) | 159.3 (4) |
| N11-H114...F13 ${ }^{\text {ii }}$ | 1.033 (5) | 1.740 (5) | 2.767 (3) | 172.2 (4) |
| N12-H121...F11 ${ }^{\text {iii }}$ | 1.021 (5) | 1.763 (5) | 2.760 (3) | 164.5 (5) |
| N12-H123..FF22 ${ }^{\text {iv }}$ | 1.017 (5) | 1.866 (6) | 2.811 (3) | 153.1 (4) |
| N12-H124...F23 ${ }^{\text {v }}$ | 1.022 (5) | 1.762 (5) | 2.767 (3) | 166.6 (3) |
| $\mathrm{N} 21-\mathrm{H} 212 \cdots \mathrm{~F} 22^{\text {vi }}$ | 1.015 (5) | 1.711 (5) | 2.708 (3) | 166.6 (5) |
| N21-H213..FF14 ${ }^{\text {vii }}$ | 1.023 (4) | 1.754 (6) | 2.777 (3) | 178.4 (4) |
| N21-H214...F11 ${ }^{\text {viii }}$ | 1.027 (5) | 1.849 (5) | 2.833 (3) | 159.1 (4) |
| N21-H214...F13 ${ }^{\text {viii }}$ | 1.027 (5) | 2.232 (5) | 2.958 (2) | 126.3 (4) |
| N22-H222 . . F12 ${ }^{\text {iii }}$ | 1.028 (5) | 1.704 (5) | 2.715 (3) | 166.1 (6) |
| N22-H223 . . F24 ${ }^{\text {iv }}$ | 1.017 (4) | 1.742 (4) | 2.757 (3) | 177.1 (4) |
| N22-H224...F21 ${ }^{\text {v }}$ | 1.016 (5) | 1.883 (6) | 2.858 (3) | 159.7 (4) |

Symmetry codes: (i) $\frac{1}{2}-x, y-\frac{1}{2}, z-\frac{1}{2}$; (ii) $\frac{1}{2}-x, \frac{1}{2}+y, z-\frac{1}{2}$; (iii) $\frac{1}{2}+x, \frac{1}{2}-y, z ;$ (iv) $1-x,-y, \frac{1_{2}^{2}}{2}+z ; \quad$ (v) $1-x, 1-y, \frac{1}{2}+z$; (vi) $x, y, 1+z ;($ vii $) \frac{1}{2}-x, y-\frac{1}{2}, \frac{1}{2}+z$; (viii) $\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z$.

Hoshino et al. (1958) there is a small spontaneous polarization $\left(0.22 \mu \mathrm{C} \mathrm{cm}^{-2}\right)$. This would correspond to a shift of $\sim 0.008 \AA$ of the ions in the cell, in good agreement with the average observed shift of $\mathrm{BeF}_{4}^{2-}$ relative to the $\mathrm{NH}_{4}^{+}$ions of $0.007 \AA$. The shifts of the individual ammonium ions virtually cancel one another.

Owing to the relative movement of the ammonium ions, and the rotations of all ions, upon going through the phase transition several hydrogen bonds become slightly shorter, while one becomes slightly longer. Thus, the N1 $\cdots \mathrm{F} 1$ distance goes from 2.779 (2) $\AA$ at 200 K to 2.805 (3) and 2.760 (3) $\AA$ in the corresponding N11 $\cdots$ F21 and N12 $\cdots$ F11 hydrogen bonds at 20 K , and

N2 . . F2 goes from 2.730 (2) A at 200 K to 2.708 (3) and 2.713 (3) $\AA$ in the corresponding N21‥F22 and N22 $\cdots$ F12 hydrogen bonds at 20 K . Thus, at 20 K the two shortest hydrogen bonds in AFB, N21...F22 2.708 (3) and N22 $\cdots \mathrm{F} 122.713$ (3) $\AA$, are both $\sim 0.02 \AA$ shorter than the shortest bond at $200 \mathrm{~K}, \mathrm{~N} 2 \cdots \mathrm{~F} 2$ 2.730 (2) $\AA$.

## 4. Conclusions

Going from the paraelectric to the ferroelectric phase, both the $\mathrm{BeF}_{4}^{2-}$ and the $\mathrm{NH}_{4}^{+}$ions rotate and shift from the mirror planes of the paraelectric phase. This results in the removal of the mirror-plane symmetry and the formation of a superlattice with the $a$ axis doubled. Along the polar $c$ axis, the $\mathrm{NH}_{4}^{+}$ions move towards the $\mathrm{BeF}_{4}^{2-}$ ions within chains of molecules and the chains move slightly relative to one another. The rotations and translations give rise to stronger hydrogen-bonding interactions. An average shift of $0.007 \AA$ of the $\mathrm{BeF}_{4}^{2-}$ ions relative to the $\mathrm{NH}_{4}^{+}$ions gives rise to the small spontaneous polarization $\left(0.22 \mu \mathrm{C} \mathrm{cm}^{-2}\right.$; Hoshino et al., 1958). Under these circumstances the $\mathrm{NH}_{4}^{+}$ions may also be expected to have some dipole moment. These ions are held in particular orientations as a result of the stronger hydrogen-bonding interactions. Any contributions to the distortions in the $\mathrm{BeF}_{4}^{2-}$ and $\mathrm{NH}_{4}^{+}$tetrahedra that are due to the anisotropic thermal motion of the atoms will be diminished at temperatures lower than the transition temperature, and the remaining distortions are then mainly due to the requirements of the hydrogen-bonding scheme. At such temperatures these tetrahedra are generally less distorted than at the transition temperature, but more so than those in the paraelectric phase.

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[^0]:    $\dagger$ The X-ray structure was first redetermined at 163 K . Details are given in the supplementary data for this paper available from the IUCr electronic archives (Reference: BK0051). Services for accessing these data are described at the back of the journal.

[^1]:    $\dagger$ The X-ray structure at 163 K also indicates similar slight distortions in both $\mathrm{BeF}_{4}^{2-}$ ions.

[^2]:    $\dagger$ Ammonium sulfate (AS) is isostructural with AFB at room temperature, where both compounds are paraelectric. The space group in each case changes from Pnma to $\mathrm{Pna}_{2}$ upon cooling into the ferroelectric phase. The transition temperature in AS is 223 K compared with 176 K in AFB and both ferroelectrics have small values of spontaneous polarization, $0.6 \mu \mathrm{C} \mathrm{cm}^{-2}$ for AS compared with $0.22 \mu \mathrm{C} \mathrm{cm}^{-2}$ for AFB. Both have small peak values for the dielectric and Curie constants at $T_{c}$. There are, however, significant differences. There is a drastic change in polarization at the transition in AS; the unit cell also changes abruptly and the crystal breaks, unless the temperature change is very slow and the crystals very thin. These changes are smooth through the transition in AFB. There is no superlattice and doubling of the $a$ axis in AS at the transition, as is the case in AFB. The specific heat shows two typical lambda-point anomalies (at 223 and 225 K ) in AS, whereas AFB shows only one anomaly at the transition. There is a double hysteresis loop in AFB just before transition, but only one such loop in AS. There is also no intermediate paraelectric phase in AS. Schlemper \& Hamilton (1966) have determined the structure of AS in the ferroelectric phase and they attribute the transition to changes in hydrogen bonding. However, it appears that it may not be possible to understand all of the differences in the physical behavior of AS and AFB purely on the basis of their respective hydrogen-bonding schemes.

